Defining and Assessing Trend Using Mark-Resight Estimates for the Number of Female Grizzly Bears with Cubs-of-the-Year in the Greater Yellowstone Ecosystem

Final Report to the Interagency Grizzly Bear Study Team

Christopher P. Peck
Department of Mathematical Sciences
Montana State University

May 11, 2016

Abstract

Population trend analysis is commonplace for researchers tasked with monitoring species and assessing the success of conservation efforts. Though the definition and estimation of a trend are linked, it is not uncommon for trends to go undefined in ecological publications. This in turn can have a deleterious effect on the transparency of statistical results informing management decisions. In an effort to shift this paradigm, we present an overview of methods used to assess trend found within ecological literature and connect them to formally defined trends. The motivating example for this paper concerns the number of female grizzly bears with cubs-of-the-year (FCOY) in the Greater Yellowstone Ecosystem (GYE) based on yearly mark-resight data. Using posterior distributions of unmarked FCOY from a Bayesian latent multinomial model, we assess the ability to detect a true decline using computer simulation under several different definitions of trend. The preliminary results presented in this paper will hopefully serve as a catalyst for further study into the complex objective of detecting declines in the GYE grizzly bear population.

1 Introduction

A trend is a trend is a trend.
But the question is, will it bend?
Will it alter its course
Through some unforeseen force,
And come to a premature end?

Sir Alexander Cairncross

Monitoring trends in population size is an essential conservation tool for long-lived, slow-reproducing taxa, which are most susceptible to over-harvest (Garshelis et al., 2006). However, there does not appear to be a consensus regarding the best way to define and assess trends in the field of ecology. This challenge is often further perpetuated by a disconnect between the definition and the method used to assess and estimate a true trend in a population measure. As a result, trends taken to be self-explanatory may inhibit peer assessment of the appropriateness of the definition and/or method of assessment. The implications of monitoring trends are often far-reaching, such as informing decisions to list or delist an animal from the threatened or endangered species list. For this reason, it is important to be as clear and transparent as possible in the definition and assessment of trends so that management decisions can be made on the best available science.

We have two objectives in this paper. First, we present an overview of methods used to assess trend found within ecological literature and connect them to formally defined trends. Second, we motivate an example concerning the number of female grizzly bears with cubs-of-the-year (FCOY) in the Greater Yellowstone Ecosystem (GYE) based on data from aerial sightings and results from yearly mark-resight analyses. Using posterior distributions of unmarked FCOY from a Bayesian latent multinomial model, we assess the ability to detect a true decline using computer simulation under several different definitions of trend.

2 General Strategies for Assessing Trend

Although not exhaustive, we present an overview of some common methods used to assess trends in population size by researchers in the field of ecology. Additionally, we will attempt to highlight each method's strengths, weaknesses, and connection to a defined trend. To avoid potential misunderstanding regarding the meaning of trend, we will hereafter reserve the term trend to mean the true trend that we wish to model/estimate. For example, we may be interested in estimating the linear component of how the population size is changing over time, even though the process is likely more complex than that.

2.1 Linear Model

The simplest of trends, the linear trend, is defined as a constant change in the response variable (e.g., yearly counts of FCOY) for a specified period of time. The linear trend is commonly estimated by the slope parameter of the least-squares line and evidence for non-zero linear trends can be assessed using a t-test if assumptions are adequately met. The method's simplicity and natural interpretation of the estimated slope parameter make it an attractive option for many researchers. Inferential objectives for this method are twofold: (i) assessing evidence for non-zero linear trend and (ii) estimating the magnitude and uncertainty of the linear trend. This method is generally applied to shorter time series (less than 15 years) but application to longer series is not uncommon (e.g., Holmes et al. (2001) investigated 30 year trends of forest birds in New Hampshire using linear regression). One potential drawback of the linear trend is the biologically unrealistic assumption of constant change in the response for the time period considered. However, this assumption may be realistic enough for the beginning years of long-term monitoring programs.

2.2 Exponential Model

The exponential growth trend is defined as a constant multiplicative change in the response variable over a specified period of time (Figure 1). This is equivalent to a so-called log-linear trend, which is defined as a constant additive change in the logarithm of the population size for a specified period of time. This reformulation allows the log-linear trend to be estimated by the slope parameter of the least-squares line after log transformation of the population size. The exponential growth trend is common in ecological literature because it is a standard model used to describe the growth of a single population in the field of population ecology. The exponential growth trend can be defined in terms of the instantaneous growth rate (λ) and this parameter can be estimated by the exponentiated slope coefficient from a log-level model fit to a specified period of time. A benefit of defining trend in terms of λ is many management officials are familiar with the parameter and its connection to population growth. However, because there are several mathematical definitions of λ in this field, it is imperative that any use of λ be accompanied by a formal definition.

The exponential growth trend also makes the potentially unrealistic assumption of constant change but this may be useful in some applications. Furthermore, both linear and exponential growth trends are commonly estimated using methods that assume observations are independent. Time series almost always violate this assumption but methods exist to account for the presence of autocorrelation. For example, Chaloupka et al. (1999) account for autocorrelated errors in the log-level model by including a second order moving average error term in their assessment of a linear trend in humpback whale abundance. By explicitly incorporating the temporal autocorrelation, the authors avoid overstating uncertainty in the estimated

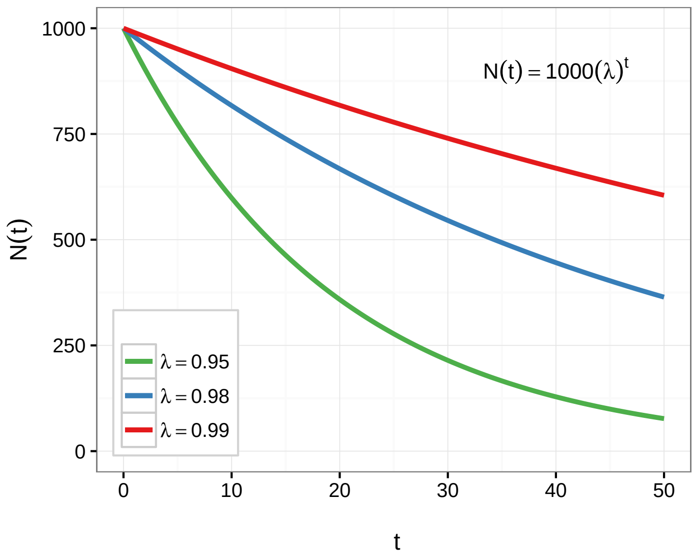

Figure 1: Example of exponential growth trend with $\lambda \in \{0.95, 0.98, 0.99\}$.

log-linear trend due to the presence of negative autocorrelation.

There are several instances of the exponential model applied to the *Ursus* genus: Swedish brown bears (Swenson et al., 1994; Kindberg et al., 2011), North American black bear (Garshelis et al., 2006), and grizzly bears in GYE (Knight et al., 1995).

2.3 Generalized Additive Model

Wood (2006) defines the generalized additive model (GAM) as a generalized linear model with a linear predictor that involves the sum of smoothed functions of covariates. GAMs are often hallowed for their flexibility and ability to avoid, either explicitly or implicitly, making assumptions about the parametric form of the function to be fitted to the time series (Crawley, 2012). That being said, there is no immediately obvious definition of trend in terms of a model parameter that can be estimated by GAMs since parameter estimates are not returned. This could prove problematic if management plans require a definition and method be explicitly stated in monitoring protocols.

Further complications arise when deciding upon the appropriate degree of smoothing. Although cross-validation methods exist to provide 'optimal' smoothness under some criteria (loss function), it is difficult to assess how the degree of smoothness affects the estimates of a defined trend. Furthermore, like all smoothers, GAMs are subject to end effects which is particularly problematic if that is the part of the series we are most interested in. Although GAMs appear to be less than ideal for the purposes of estimating trends, they can be useful as a statistical tool used to explore potential functional relationships in the response. For example, Donner et al. (2008) use a GAM for the purposes of a change point analysis where they identified key points of change in the gradient of fitted values to describe changes in the population of Kirtland's warblers over time.

The recent application of GAMs in ecological literature has primarily been in the field of ornithology: farmland birds in the United Kingdom (Fewster et al., 2000; Freeman et al., 2001), male Kirtland's warblers in Michigan (Donner et al., 2008), and waterbirds in Great Britain and Northern Ireland (Atkinson et al., 2006). Additionally, Balazs et al. (2004) present a Bayesian GAM model to assess trends in Hawaiian green sea turtle nester abundance. All four analyses involve time series at least 25 years long and Atkinson et al. (2006) state the method "relies on extensive counts made over long-periods, a situation of data-richness that is unlikely to exist beyond a few NW European countries."

2.4 Additional Approaches

This section serves to provide a summary of additional modelling approaches found in the ecological literature, particularly those applied to the *Ursus* genus. We do not attempt to review any of these additional methods or connect them to a defined trend at this time. However, they may serve as a convenient starting-point for future efforts aimed at defining and assessing trend.

- Wiener-drift process yielding a lognormal probability distribution of population abundance (Dennis et al., 1991)
- Overview of using both MONITOR and TRENDS power analysis programs (Hatch, 2003) and Poisson regression using TRIM software package (Conrad et al., 2006)
- Kalman-filter framework of structural time-series models (Visser, 2004) and applied use of TrendSpotter software (developed by H. Visser) to waterbird monitoring data (Soldaat et al., 2007)
- Theil's non-parametric test based on log-mean count annual indices (Morrison et al., 1994)
- Pradel models implemented in MARK for DNA mark-recapture data from grizzly bears in Northern Continental Divide Ecosystem (Stetz et al., 2010) and British Columbia (Boulanger et al., 2004)
- Spatial and temporal trends in harbour porpoise using MCMCglmm in R (Peschko et al., 2016)
- Finite rate of increase ($\lambda = N_{t+1}/N_t$) using a revised Lotka equation for grizzly bears in the Swan Mountains, MT (Mace et al., 1998) and Yellowstone (Eberhardt et al., 1994)
- 3-year moving average using Bayesian latent multinomial model (Higgs et al., 2013)

2.5 General Oversights

Regardless of the definition of trend, it is not uncommon for researchers to fail to keep reference to the number of years the trend is defined for. This is particularly important when estimating short-term trends because the number of data points informing the estimate can be influential. When comparing results from several definitions is of interest, failure to maintain this reference may lead to unwarranted or misleading comparisons.

3 Motivating Example

Management and government officials alike are interested in both the growth rate of GYE grizzly bears in the past and how the population is changing now (Harris et al., 2007). Formed in 1973, the Interagency Grizzly Bear Study Team (IGBST) is an interdisciplinary team of researchers responsible for research efforts on GYE grizzly bears. Under the Grizzly Bear Recovery Plan (USFWS, 1993), IGBSTs monitoring program includes both annually estimating the number of FCOY in the GYE population and assessing trend for this segment of the population (IGBST, 2014). FCOY are an easily recognizable cohort and changes in population size for this segment of the population will generally track changes for the population as a whole.

3.1 Current Protocol for Estimating FCOY

IGBST (2014) outlines the protocol for estimating population size from counts of unique FCOY in a given year. First, the number of unique FCOY from ground observations and aerial sightings is estimated using a rule set developed by Knight et al. (1995). This estimate of unique females provides a minimum annual estimate of FCOY in the population (Cherry et al., 2007). Using only FCOY observed within the Demographic Monitoring Area (Figure 2), the Chao2 estimator (Wilson et al., 1992; Keating et al., 2002) is applied to sighting frequencies estimated by the rule set for each unique family. The result is an estimate of the total FCOY present in the population. It should be noted that simulation studies indicate that the rule set of Knight et al. (1995) inherently underestimates known numbers of unique FCOY (Schwartz et al., 2008), which makes sense given that it was meant to estimate a minimum. Furthermore, simulations suggest an additional, but smaller, source of underestimation bias comes from the Chao2 estimator itself (Wilson et al., 1992; Keating et al., 2002; Cherry et al., 2007).

3.2 Mark-Resight Method

In an attempt to obtain more reliable inference for FCOY abundance, Higgs et al. (2013) propose a mark-resight approach utilizing a Bayesian latent multinomial

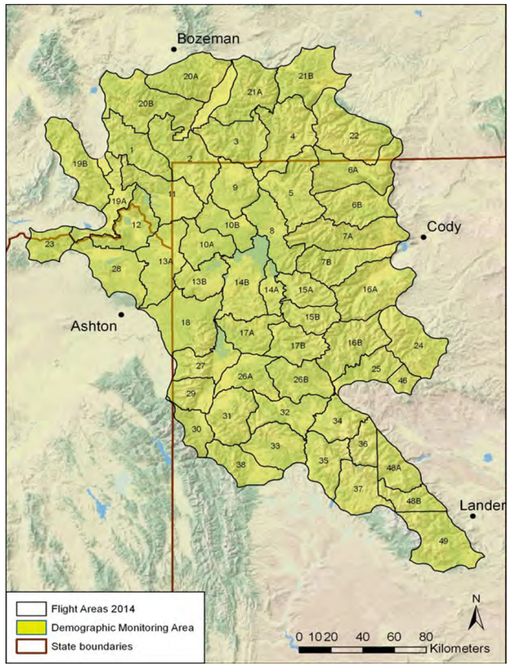

Figure 2: According to IGBST (2014), only FCOY observed within the Demographic Monitoring Area will be used for population estimation.

model for inference. IGBST has conducted two standardized observation flights within Bear Observation Areas (Figure 2) per year since 1997. Each year, the number of marked FCOY sighted zero, one, and two times, and the total number of sightings of unmarked FCOY are recorded. Using a common sightability model, assuming homogeneity in sighting probabilities over all previous sampling occasions (years), and data obtained from aerial surveys each year, we obtain a yearly posterior distribution for population size of unmarked FCOY.

3.3 Research Question

The IGBST has identified the years 2002-2015 of particular interest. This decision was motivated by demographic analyses which indicated a slowing of population growth due to changes in vital rates during 2002-2011 compared to 1983-2001 (IGBST, 2012). The study team is interested in examining the following question regarding the application of the mark-resight estimator: assuming that the population of unmarked FCOY has been stable for the years of 2002-2015 (the so-called 'plateau phase'), can we detect a decline in the population of {1%, 2.5%, 5%} per year after {5, 10, 15, 20} years time? To address this question, we will begin by providing several potential definitions of trend followed by an assessment of our ability to detect a decline in the number of FCOY in the population using simulation.

3.4 Defining Trend

Before proceeding with any analyses, we will present several definitions of trend and justify the appropriateness of their application to FCOY monitored by the IGBST. Note that all of the following definitions represent different mathematical definitions of change in population size. As introduced in Section 2.2, the exponential growth model is commonly used in ecology to model the growth (and decline) of populations. Furthermore, the instantaneous rate of population change is the parameter of interest in IGBSTs current trend assessment protocol (IGBST, 2014).

> **Definition 1.** instantaneous rate of FCOY population change (λ) assuming exponential growth over a specified number of years

An important facet of this definition to keep in mind, and all remaining definitions, is the explicit number of years for which trend is defined. The exponential growth model can be expressed mathematically as

$$N_{u,t} = N_{u,1} \cdot \lambda^t, \tag{1}$$

where $N_{u,t}$ is equal to the number of unmarked FCOY at time t. Taking the natural logarithm of each side yields

$$\ln(N_{u,t}) = \ln(N_{u,1}) + t \cdot \ln(\lambda). \tag{2}$$

The structural form of Equation 2 indicates that λ may be estimated by the exponentiated slope coefficient from the least-squares regression of $\ln(N_{u,t})$ on t. It should be noted that this is not the only method available for estimating this definition of λ but it is the method that we will proceed with. A direct result of the definition is λ values less than 1 imply the population is experiencing decline. Therefore, we will be interested in computing the posterior probability of $\lambda < 1$ for each time frame considered.

Harris et al. (2006) consider two alternative definitions of the 'population trajectory' summary statistic λ:

> **Definition 2.** the geometric mean of the n ratios of unmarked FCOY in $n+1$ successive years

> **Definition 3.** the arithmetic mean of the n ratios of unmarked FCOY in $n+1$ successive years

Both definitions define the ratio of unmarked FCOY in successive years in terms of the finite rate of increase

$$\lambda_f(t) = \frac{N_{u,t+1}}{N_{u,t}}, \tag{3}$$

which assumes exponential growth for each successive 1 year period. The geometric mean of n ratios of unmarked FCOY in $n+1$ successive years can be computed as

$$\bar{\lambda}_2 = \left(\prod_{i=1}^{n} \frac{N_{u,\,i+1}}{N_{u,\,i}} \right)^{\frac{1}{n}} = \left(\frac{N_{u,\,n+1}}{N_{u,\,1}} \right)^{\frac{1}{n}}. \tag{4}$$

Note that the geometric mean can be alternatively expressed as the exponentiated arithmetic mean of the log-transformed ratios of unmarked FCOY

$$\bar{\lambda}_2 = \exp\left[\frac{1}{n} \sum_{i=1}^{n} \ln\left(\frac{N_{u,\,i+1}}{N_{u,\,i}} \right) \right]. \tag{5}$$

Additionally, the arithmetic mean for n ratios of unmarked FCOY in $n+1$ successive years can be computed as

$$\bar{\lambda}_3 = \frac{1}{n} \sum_{i=1}^{n} \frac{N_{u,\,i+1}}{N_{u,\,i}}. \tag{6}$$

For either estimate, we are interested in computing the posterior probability of $\bar{\lambda} < 1$ for each time frame considered. By taking the geometric or arithmetic mean of the λ_f's, we obtain a measure of average finite rate of increase for the time period under consideration. This can be thought of as an extreme case of smoothing over the specified time period.

Instead of assuming exponential growth, we can alternatively consider the annual linear rate of change in population size.

> **Definition 4.** an annual rate of linear FCOY population change over a specified number of years

We will investigate the annual linear rate of population change as estimated by the least-squares regression slope coefficient (β_1) of $N_{u,\,t}$ on t. Given this definition, we will be interested in the posterior probability of $\beta_1 < 0$ for each time frame considered.

4 Methods

4.1 Simulating FCOY

In order to simulate realizations of FCOY consistent with mark-resight sampling, we will use R (R Core Team, 2016) and the `griz_sim_Fcoy()` function from the `grizzly` package written by Michael Lerch for IGBST. The user must provide the function with the following arguments:

- `unmarked_total`: vector of true number of unmarked FCOY in each year
- `marked_total`: vector of true number of marked FCOY in each year

- `detect_prob`: sighting probabilities $\boldsymbol{\pi} = (\pi_0, \pi_1, \pi_2)$ where π_0, π_1, and π_2 represent the probability that an unmarked FCOY is sighted 0 times, 1 time, and 2 times in a given year.

To define `unmarked_total`, we assume the population of FCOY is not changing from 2002-2015 (i.e., the 'plateau phase') followed by a decrease of (1%, 2.5%, 5%) per year (rounded to nearest integer) for 20 years. The simulated true number of unmarked FCOY during the period of 2002-2015 was set to 70, which seems reasonable given the posterior distributions of unmarked FCOY during the period 2002-2012 (Figure A.1) reported in Table 2 of Higgs et al. (2013). Furthermore, assuming a total of 70 FCOY resulted in simulated numbers of sightings similar to those observed in practice.

To define `marked_total`, we assume that IGBST maintains a constant year-to-year count of marked FCOY. The maximum number of marked FCOY for the entire region (excluding moth aggregation sites) for years 2002-2014 was 10 (Higgs et al., 2013), which we use as a "best-case scenario". Finally, we define `detect_prob` to be equal to the sighting probabilities specified by IGBST: $\boldsymbol{\pi} = (0.70, 0.25, 0.05)$, which approximates the proportion of marked bears observed zero, one, and two times over all years since 1997.

4.1.1 Simulation Assumptions

Given that all results derived from this simulation are dependent on the assumptions made in simulating the hypothetical realizations, we take this opportunity to explicitly summarize and justify the assumptions for the reader.

First, the sighting probabilities $\boldsymbol{\pi}$ are assumed to be constant year-to-year and identical for both marked and unmarked FCOY. In practice, sighting information is pooled across years since the information from any given year is very limited given the relatively small number of marked FCOY and the low sighting probability. Second, the number of marked FCOY remains constant for the entire time period considered. Although a simplifying assumption, we believe it is reasonable since the number of marked females has remained relatively constant over time despite changes in the population size. Third, we simulate a multiplicative decline in unmarked FCOY using the following equation

$$N_{u,t} = 70 \cdot \lambda^t \qquad t = 1, \ldots, 20\,, \tag{7}$$

where $N_{u,t}$ is rounded to the nearest integer and $\lambda \in \{0.99, 0.975, 0.95\}$. Assuming a multiplicative decline in the population of unmarked FCOY is consistent with current IGBST methods of assessing trend in FCOY. However, Equation 7 intrinsically assumes the multiplicative decline is constant across years. While this is not a realistic assumption, it is necessary to assess ability to detect a decline of {1%, 2.5%, 5%} per year after {5, 10, 15, 20} years time.

Typically when considering trend, we are simulating a decline in the population size due to death of individuals. The FCOY trend is not only based on annual survival, but also on annual reproduction rates. Furthermore, female grizzly bears reproduce, on average, every three years. Therefore, a bear who is an FCOY one year may still be alive the following year, but unlikely as an FCOY. To remedy this, we could simulate a decline in the number of female grizzly bears each year through the use of a Markov process with transition probabilities for mortality and reproduction. Sighting probabilities could then be sampled each year from Dirichlet(62.5, 18.5, 1.5) (Higgs et al., 2013) which would be applied to the subset of females that have cubs. Although there are numerous ways to simulate more realistic data, we believe the results presented in this paper represent a useful starting point.

4.2 Estimating FCOY

We used the Gibbs sampler outlined in Section 3.3 of Higgs et al. (2013) to estimate the number of unmarked FCOY using the realizations of simulated FCOY from Section 4.1. An implementation of the Gibbs sampler is available via the `griz_Fcoy_sampler()` function in the `grizzly` package. For each realization, the sampler was run for 100,000 iterations, saving every fifth iteration. Preliminary runs indicated the number of iterations was sufficient to feel comfortable with convergence, as assessed by the Gelman and Rubin Statistic $\widehat{R}$ and visual inspection of the traceplots. The posterior distributions of unmarked FCOY for each year ($N_{u,t}$) are then obtained after removing the first 500 saved draws, for a total of 19,500 used to approximate each posterior distribution.

4.3 Posterior Distribution of Estimated Trend

As stated previously, we obtain posterior distributions of unmarked FCOY for each year of the simulation. Given the posterior distributions for N_u, the posterior distribution for $g(N_u)$ is completely specified (Link et al., 2009). In other words, we can obtain a posterior distribution of the estimated trend so long as it is defined as a function of the posterior distributions for N_u.

Using the ith posterior draw from each independent posterior distribution of unmarked FCOY, we obtain a time series of length 20 years. Using all posterior draws yields 19,500 time series where each individual series can be interpreted as a series of point estimates of yearly unmarked FCOY that could have resulted from the realization of simulated FCOY. To each individual series, a summary measure $g(N_u)$ used to estimate a defined trend will be computed for varying lengths of time (e.g., first five years, first ten years, etc). Saving the estimated parameter from each draw results in a posterior distribution for that parameter for the specified number of years. From this posterior distribution we can compute the posterior probability of interest related to population decline.

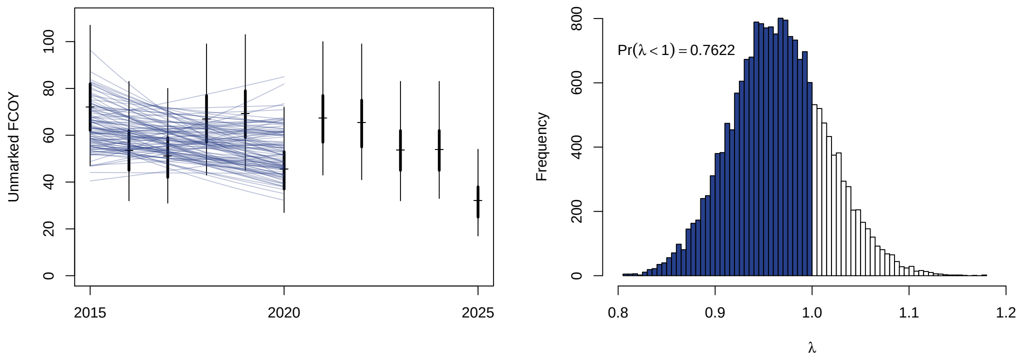

Figure 3: (Left) Posterior distributions of N_u for the first 10 years of data simulated under a 1% yearly reduction in population size beginning with 70 unmarked FCOY. Back-transformed log-level regression lines displayed for a random sample of 100 posterior draws. (Right) Posterior distribution of λ defined for the first 5 years with posterior probability of $\lambda < 1$ displayed.

4.3.1 Posterior of Derived Parameter Example

Here, we outline the steps taken to obtain the posterior distribution of λ for a realization of 20 years of data under a 1% yearly reduction in population size beginning with 70 unmarked FCOY. Figure 3 displays the posterior distributions of N_u from the latent multinomial model for the first 10 simulated years. For this example, we will focus on λ defined for the first 5 years and fit a log-level regression to every 5-year series of posterior draws of N_u. The back-transformed log-level regression lines for a random sample of 100 posterior draws are displayed in Figure 3. For each log-level regression, we compute the derived parameter λ as the exponentiated slope coefficient. Collectively, all 19,500 derived values of λ make up the posterior distribution of λ (Figure 3) from which posterior quantities of interest (e.g., $\Pr(\lambda < 1)$) can be readily computed.

5 Results

5.1 Exponential Rate of Change (λ)

We observe, as expected, our ability to detect the true decline increases both with increasing number of years of decline considered and increasing percent decline per year. From a practical management perspective, IGBST is interested in the ability to detect declines after at most 10 years. There is poor ability to detect declines after 5 years, regardless of the yearly percent decline, as evidenced by the wide range of posterior probabilities (Figure 4). Considering 10 years also indicates poor ability

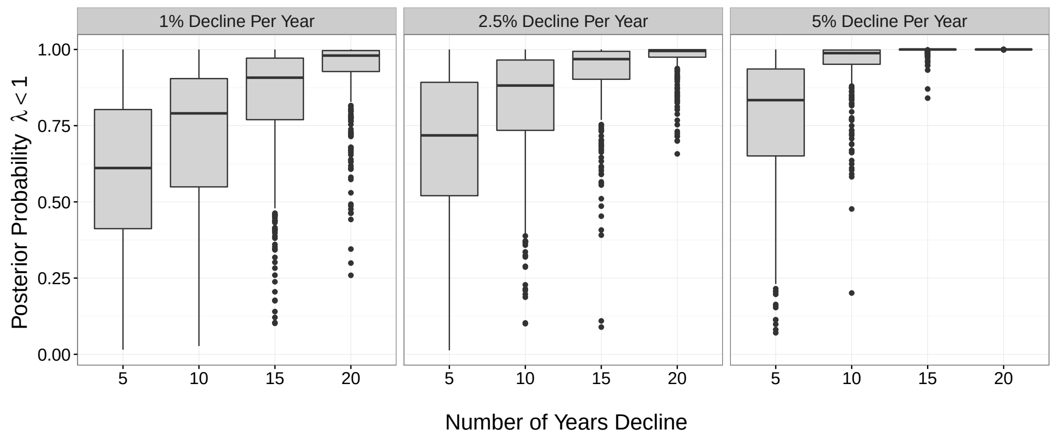

Figure 4: Boxplots of posterior probabilities of $\lambda < 1$ for 500 realizations at each specified percent decline and t years of decline.

to detect declines of 1% and 2.5% per year and moderate ability for a 5% per year decline.

5.2 Geometric Mean ($\bar{\lambda}_2$)

We observe wider ranges of posterior probabilities compared to the posterior probabilities for λ (Figure 4). Ability to detect a true decline after 5 or 10 years is poor, regardless of the percent decline considered.

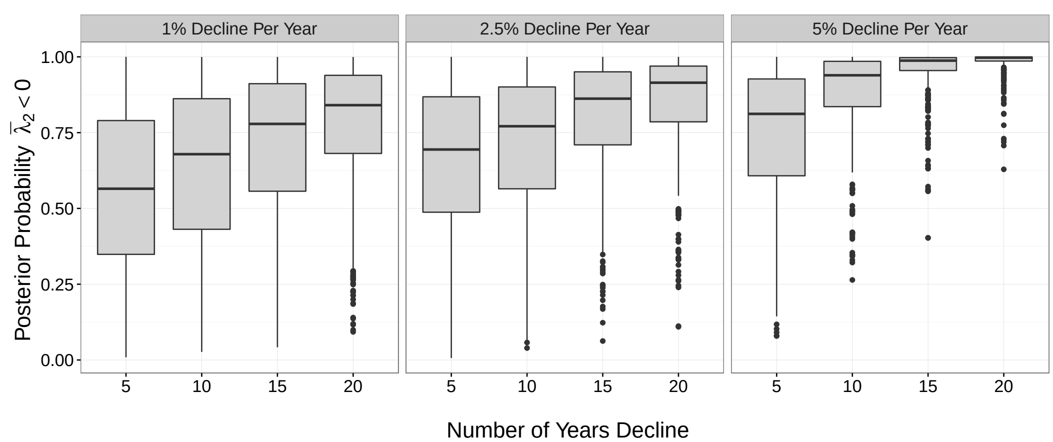

Figure 5: Boxplots of posterior probabilities of $\bar{\lambda}_2 < 1$ for 500 realizations at each specified percent decline and years of decline.

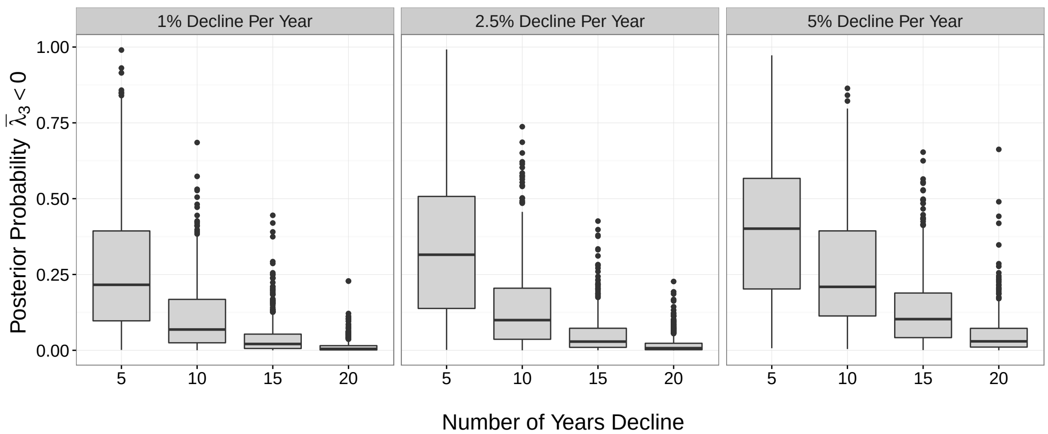

Figure 6: Boxplots of posterior probabilities of $\bar{\lambda}_3 < 1$ for 500 realizations at each specified percent decline and years of decline.

5.3 Arithmetic Mean ($\bar{\lambda}_3$)

It is immediately apparent that the ability of $\bar{\lambda}_3$ to detect a true decline is extremely poor. Additionally, we observe that increasing the number of years considered further decreases the detection accuracy. While seemingly counterintuitive, this result is a product of the year-to-year variability inherent in the posterior distributions. For example, consider the random sample of 20 posterior draws from one simulated realization (Figure A.2). It is clear that some yearly ratios result in values much greater than 1, which in turn inflates $\bar{\lambda}_3$. By considering longer periods of decline, we in turn increase the instances of ratios greater than 1 which lowers our ability to detect decline based on the posterior probability of $\bar{\lambda}_3 < 1$.

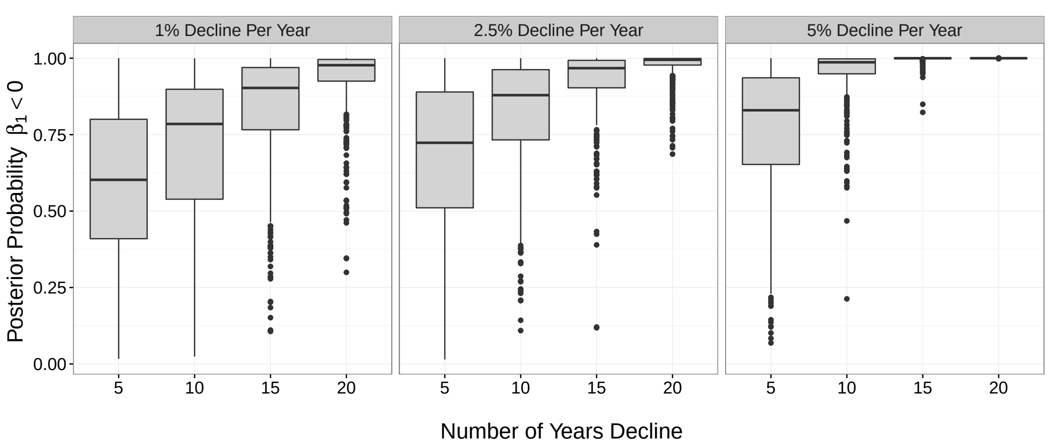

Figure 7: Boxplots of posterior probabilities of $\beta_1 < 0$ for 500 realizations at each specified percent decline and t years of decline.

5.4 Linear Rate of Change (β_1)

Again, we observe poor ability to detect declines after 5 years, regardless of the yearly percent decline. The 10 year period also indicates poor ability for declines of 1% and 2.5% per year and moderate ability for a 5% per year decline. The exponential trend underlying the simulation is not distinguishable from linear due to the lack of curvature, so both λ and β_1 definitions are nearly identical in their ability to detect the true decline.

5.5 Direct Comparisons

As mentioned previously, the ability to detect declines after 5 or 10 years is practically meaningful from a management perspective. Accordingly, we present a more direct comparison of detection ability among the mathematical definitions of trend considered (Figure 8).

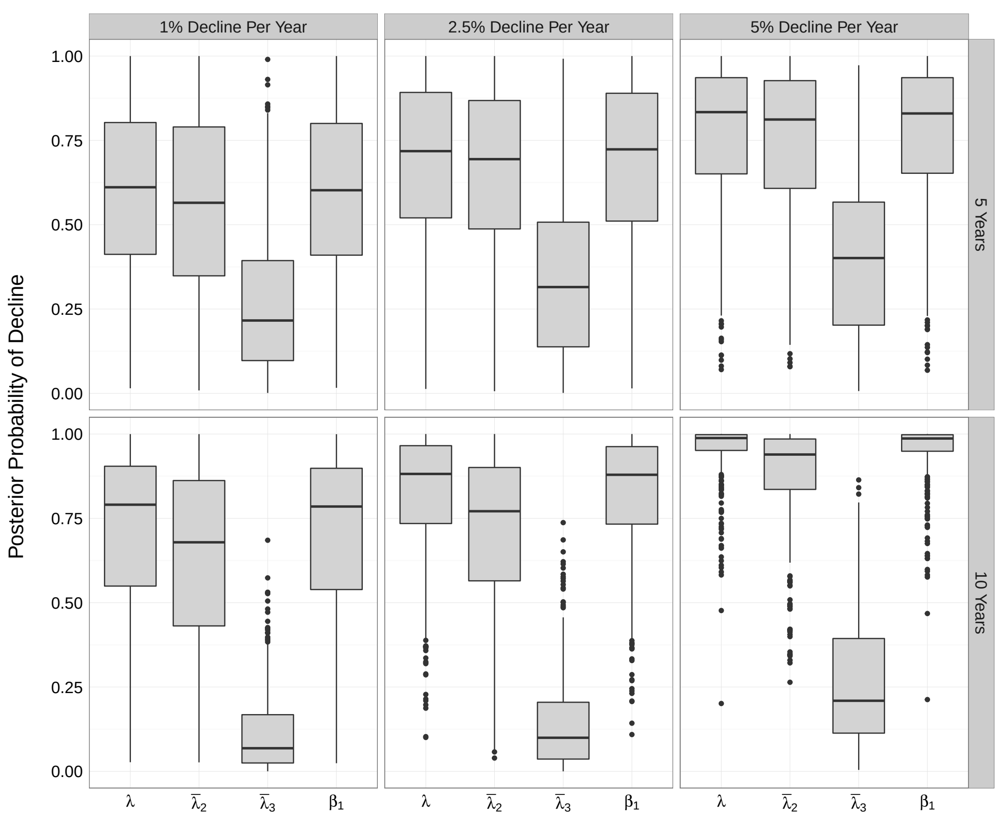

Figure 8: Comparison of posterior probabilities of decline across mathematical definitions of trend. Each boxplot contains 500 realizations at each specified percent decline (columns) and years of decline (rows).

6 Discussion

6.1 Connections to Traditional Power Curves

Had we considered some cutoff value c indicative of ability to capture true decline (e.g., $c = 0.9$), we could compute the number of realizations out of 500 resulting in a posterior probability greater than c. This would result in curves similar to traditional 'power curves' in each of the facets of Figures 4–6. However, we chose not to disseminate the results in this manner for two reasons. First, the choice of the cutoff value c is arbitrary (see Section 6.2 for discussion). Second, by observing the raw posterior probabilities, we can visually assess the variability in our ability to detect trend. For example, consider the posterior probabilities from two hypothetical definitions of trend, each containing 33 realizations (Figure 9). If we consider the cutoff $c = 0.85$, we would conclude that both have similar ability to detect a true decline since both have 11 out of 33 realizations that were above $c = 0.85$. However, it is clearly evident that the left boxplot is inferior in its ability to detect a true decline due to its variability as compared to the right boxplot. For this reason, we decided the raw posterior probabilities are not only more useful in assessing ability to detect trend, but it also better aligns with our belief in transparency. We do, however, recognize that a cutoff may have to be implemented in order to make management decisions.

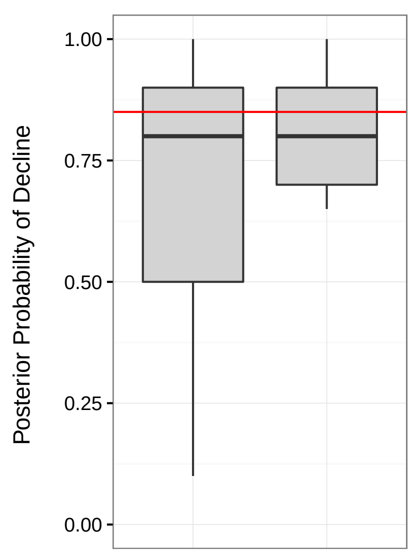

Figure 9: Hypothetical situation in which assessment of ability to detect trend may be less informative if we consider the number of realizations above the cutoff value $c = 0.85$ (red horizontal line) instead of observing the raw posterior probabilities.

6.2 Cutoff Value Considerations

The choice of a cutoff value c is technically arbitrary. However, this does not imply that the selection of a cutoff value should be made without considerable thought concerning its implications from a management perspective. For example, we can think about each posterior draw as a hypothetical mark-resight time series of N_u that could be observed given the observed sighted counts. We can then ask questions such as "what proportion of the 19,500 hypothetical time series would need to indicate a decline for managers to feel comfortable or confident concluding the population is in decline?"

6.3 Autocorrelation

Since the data were simulated in the absence of autocorrelation and the latent multinomial referenced in Section 4.2 obtains results for each year separately, we did not pursue time dependence in our estimation of trend. Additionally, an ACF plot of the observed FCOY counts during 1997–2012 does not indicate strong autocorrelation within the series (Figure A.3).

This naturally begs the question of whether or not incorporating autocorrelation into the estimate of a defined trend is appropriate in this particular application. If positive autocorrelation is expected, then incorporating autocorrelation will improve ability to detect the trend. However, it is important to remember that in these simulations we are introducing an unrealistic process (see Section 4.1.1). Therefore, the combination of not accounting for autocorrelation and having an unrealistic process might balance each other out somehow. Regardless, we encourage a thorough investigation of trend detection sensitivity in the presence of time dependence in future work. In particular, the implications of both positive and negative autocorrelation on more realistic simulations of the underlying process should be explored.

6.4 Additional Considerations

The practicality of the results presented in Section 5 are constrained by the assumptions made in Section 4.1.1. Lack of verity to the true underlying process of female grizzly bear reproduction and large uncertainty in posterior distributions of N_u pose the greatest impediments.

The general lack of ability to detect a true decline can be primarily attributed to the amount of uncertainty present in the posterior distributions of unmarked FCOY. A primary source of uncertainty in the posterior for N_u is the uncertainty in the estimates of the sighting probabilities $\boldsymbol{\pi}$. This is largely due to the low number of marked females in the population. However, increasing the number of marked females is a non-trivial task subject to many logistical and bureaucratic constraints.

7 Conclusion

In this paper, we summarized common methods used to assess trends in population size and connected them to formally defined trends. As a motivating example, we provided an assessment of trend detection ability based on several different definitions using simulated realizations of FCOY consistent with mark-resight sampling. We observed poor ability to detect a true decline within a practically meaningful time frame across all definitions of trend considered. This, however, represents only a preliminary investigation of the ability to detect trends in FCOY within the GYE. We hope this paper serves as a catalyst for further study of this complex research objective.

References

Atkinson, Philip W et al. (2006). "Identifying declines in waterbirds: the effects of missing data, population variability and count period on the interpretation of long-term survey data". In: *Biological Conservation* 130.4, pp. 549–559.

Balazs, George H et al. (2004). "Thirty-year recovery trend in the once depleted Hawaiian green sea turtle stock". In: *Biological Conservation* 117.5, pp. 491–498.

Boulanger, John et al. (2004). "Monitoring of grizzly bear population trends and demography using DNA mark-recapture methods in the Owikeno Lake area of British Columbia". In: *Canadian Journal of Zoology* 82.8, pp. 1267–1277.

Chaloupka, Milani et al. (1999). "Estimating seasonal abundance trends and survival probabilities of humpback whales in Hervey Bay (east coast Australia)". In: *Marine ecology. Progress series* 184, pp. 291–301.

Cherry, Steve et al. (2007). "Evaluating estimators of the numbers of females with cubs-of-the-year in the Yellowstone grizzly bear population". In: *Journal of agricultural, biological, and environmental statistics* 12.2, pp. 195–215.

Conrad, Kelvin F et al. (2006). "Rapid declines of common, widespread British moths provide evidence of an insect biodiversity crisis". In: *Biological Conservation* 132.3, pp. 279–291.

Crawley, Michael J (2012). *The R book*. John Wiley & Sons.

Dennis, Brian et al. (1991). "Estimation of growth and extinction parameters for endangered species". In: *Ecological monographs* 61.2, pp. 115–143.

Donner, Deahn M et al. (2008). "Influence of habitat amount, arrangement, and use on population trend estimates of male Kirtlands warblers". In: *Landscape Ecology* 23.4, pp. 467–480.

Eberhardt, LL et al. (1994). "Population trend of the Yellowstone grizzly bear as estimated from reproductive and survival rates". In: *Canadian Journal of Zoology* 72.2, pp. 360–363.

Fewster, Rachel M et al. (2000). "Analysis of population trends for farmland birds using generalized additive models". In: *Ecology* 81.7, pp. 1970–1984.

Freeman, SN et al. (2001). "Statistical analysis of an indicator of population trends in farmland birds". In: *BTO Research Report*.

Garshelis, David L et al. (2006). "State and provincial estimates of American black bear numbers versus assessments of population trend". In: *Ursus* 17.1, pp. 1–7.

Harris, Richard B et al. (2006). "Trajectory of the Yellowstone grizzly bear population under alternative survival rates". In: *Wildlife Monographs* 161, p. 44.

Harris, Richard B et al. (2007). "Population growth of Yellowstone grizzly bears: uncertainty and future monitoring". In: *Ursus* 18.2, pp. 168–178.

Hatch, Scott A (2003). "Statistical power for detecting trends with applications to seabird monitoring". In: *Biological Conservation* 111.3, pp. 317–329.

Higgs, Megan D et al. (2013). "Insights into the latent multinomial model through mark-resight data on female grizzly bears with cubs-of-the-year". In: *Journal of Agricultural, Biological, and Environmental Statistics* 18.4, pp. 556–577.

Holmes, Richard T et al. (2001). "Thirty-year bird population trends in an unfragmented temperate deciduous forest: importance of habitat change". In: *The Auk* 118.3, pp. 589–609.

IGBST (2012). *Yellowstone Grizzly Bear Investigations: Annual Report of the Interagency Grizzly Bear Study Team.* Tech. rep. Interagency Grizzly Bear Study Team, U.S. Geological Survey, Northern Rocky Mountain Science Center, Bozeman, Montana, USA.

— (2014). *Yellowstone Grizzly Bear Investigations: Annual Report of the Interagency Grizzly Bear Study Team.* Tech. rep. Interagency Grizzly Bear Study Team, U.S. Geological Survey, Northern Rocky Mountain Science Center, Bozeman, Montana, USA.

Keating, Kim A et al. (2002). "Estimating numbers of females with cubs-of-the-year in the Yellowstone grizzly bear population". In: *Ursus*, pp. 161–174.

Kindberg, Jonas et al. (2011). "Estimating population size and trends of the Swedish brown bear Ursus arctos population". In: *Wildlife Biology* 17.2, pp. 114–123.

Knight, Richard R et al. (1995). "Appraising status of the Yellowstone grizzly bear population by counting females with cubs-of-the-year". In: *Wildlife Society Bulletin (1973-2006)* 23.2, pp. 245–248.

Link, William A et al. (2009). *Bayesian inference: with ecological applications*. Academic Press.

Mace, Richard D et al. (1998). "Demography and population trend of grizzly bears in the Swan Mountains, Montana". In: *Conservation Biology*, pp. 1005–1016.

Morrison, RIG et al. (1994). "Population trends of shorebirds on fall migration in eastern Canada 1974-1991". In: *The Wilson Bulletin*, pp. 431–447.

Peschko, Verena et al. (2016). "Trends of harbour porpoise (Phocoena phocoena) density in the southern North Sea". In: *Ecological Indicators* 60, pp. 174–183.

R Core Team (2016). *R: A Language and Environment for Statistical Computing*. R Foundation for Statistical Computing. Vienna, Austria. URL: `https://www.R-project.org/`.

Schwartz, Charles C et al. (2008). "Evaluation of rules to distinguish unique female grizzly bears with cubs in Yellowstone". In: *The Journal of Wildlife Management* 72.2, pp. 543–554.

Soldaat, Leo et al. (2007). "Smoothing and trend detection in waterbird monitoring data using structural time-series analysis and the Kalman filter". In: *Journal of ornithology* 148.2, pp. 351–357.

Stetz, Jeffrey B et al. (2010). "Evaluation of bear rub surveys to monitor grizzly bear population trends". In: *The Journal of Wildlife Management* 74.4, pp. 860–870.

Swenson, Jon E et al. (1994). "Size, trend, distribution and conservation of the brown bear Ursus arctos population in Sweden". In: *Biological conservation* 70.1, pp. 9–17.

USFWS (1993). "Grizzly bear recovery plan". In: *US Fish and Wildl. Serv., Missoula, Mont. 181pp.*

Visser, Hans (2004). "Estimation and detection of flexible trends". In: *Atmospheric Environment* 38.25, pp. 4135–4145.

Wilson, Richard M et al. (1992). "Capture-recapture estimation with samples of size one using frequency data". In: *Biometrika* 79.3, pp. 543–553.

Wood, Simon (2006). *Generalized additive models: an introduction with R*. CRC press.

A Supplementary Figures

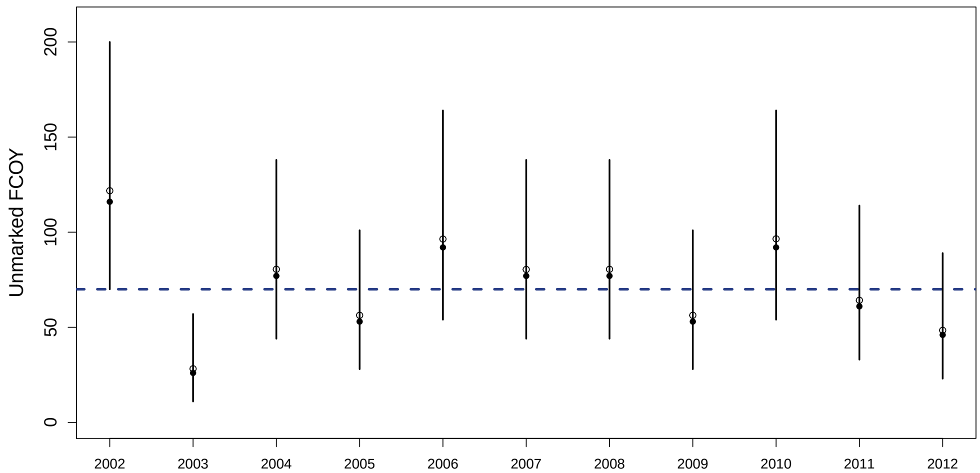

Figure A.1: Posterior means (open circles) and medians (closed circles) for total unmarked FCOY from the latent multinomial model based on data presented in Table 2 of Higgs et al. (2013). Horizontal dashed line set at 70 unmarked FCOY.

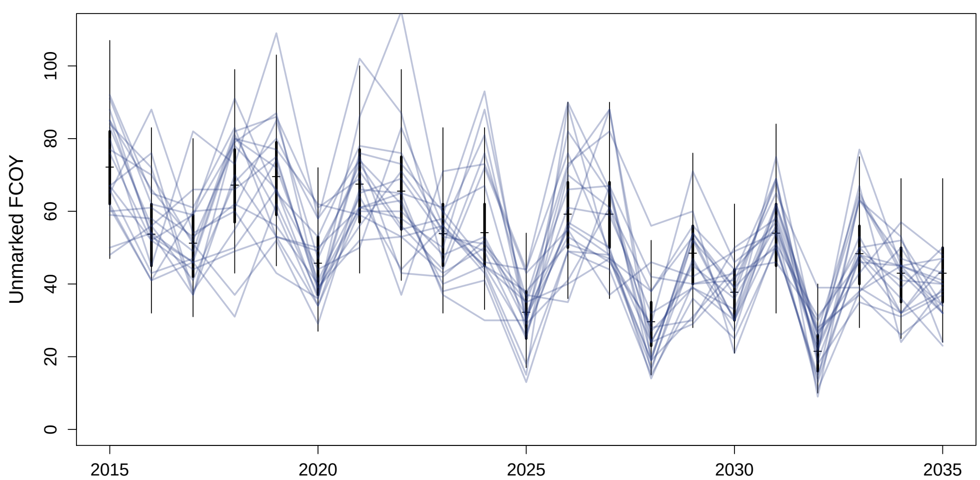

Figure A.2: Traces of 20 random posterior draws for a realization of 20 years of data under a 1% yearly decline in population size beginning with 70 unmarked FCOY. Each trace comprises the ith draw from each posterior distribution connected across the 20 year time period.

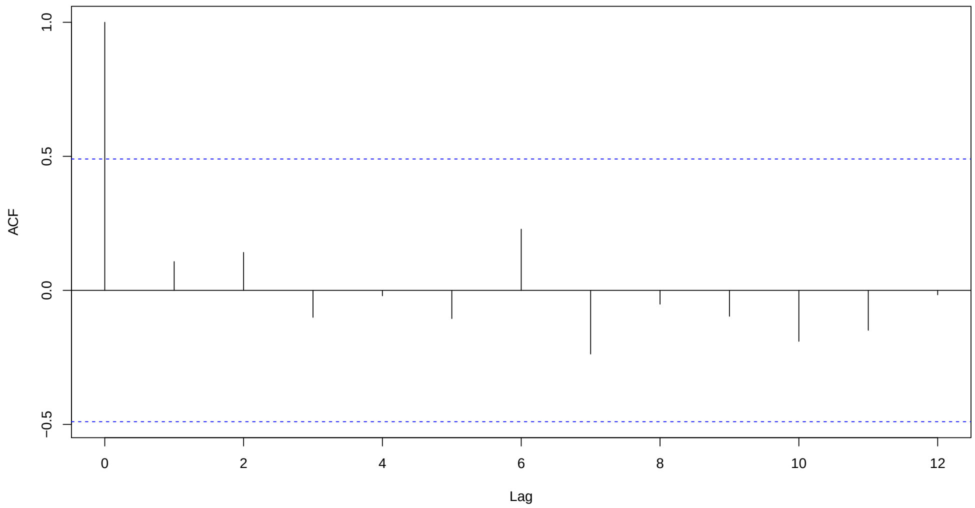

Figure A.3: ACF plot of observed FCOY counts (marked + unmarked) during 1997–2012 as given in Table 1 of Higgs et al. (2013).